2 Signals

2.1 Introduction

This chapter outlines how electrical signals are analysed.

2.2 Electrical signals

Any time-varying quantity can be represented either as a time-varying signal, or, in terms of frequencies. For example, when we describe sound we typically refer to it in terms of frequencies, such as an A defined as 440 Hz. Table 2.1 illustrates some of the frequencies for musical notes for given octaves. It can be seen that an A, in the middle C octave, has a frequency of 440Hz. One octave down from this, it is half the frequency (220Hz), and in the octave above this, it is double (880Hz).

Any electrical signal can be analysed either in the time domain, or in the frequency domain. A time-varying signal contains a range of frequencies. If the signal is repetitive (that is, it repeats after a given time) then the frequencies contained in it will also be discrete.

Table 2.1 Note frequencies (Hz) for different octaves

	Octave 1	Octave 2	Octave 3	Octave 4	Octave 5	Octave 6	Octave 7
C	32.70	65.41	130.81	261.63	523.25	1046.50	2093.00
C#,Db	34.65	69.30	138.59	277.18	554.36	1100.73	2217.46
D	36.71	73.42	146.83	293.66	587.33	1174.66	2349.32
D#,Eb	38.89	77.78	155.56	311.13	622.25	1244.51	2489.02
E	41.20	82.41	164.81	329.63	659.26	1318.51	2367.02
F	43.65	87.31	174.61	349.23	698.46	1396.91	2637.02
F#,Gb	46.25	92.45	185.00	369.99	739.99	1474.98	2959.96
G	49.00	98.00	196.00	392.00	783.99	1567.98	3135.96
G#,Ab	51.91	103.83	207.65	415.30	830.61	1661.22	3322.44
A	**55.00**	**110.00**	**220.00**	**440.00**	**880.00**	**1760.00**	**3520.00**
A#,Bb	58.27	116.54	233.08	466.16	932.33	1664.66	3729.31
B	61.74	123.47	246.94	493.88	987.77	1975.53	3951.07

The standard form of a single frequency signal is:

$$V(t) = V\sin(2\pi ft + \theta)$$

where $v(t)$ is the time-varying voltage (V), V is the peak voltage (V), f the signal frequency (Hz) and θ its phase (°).

A signal in the time domain is a time-varying voltage, whereas in the frequency domain it is voltage amplitude against frequency. Figure 2.3 shows how a single frequency is represented in the time domain and the frequency domain. It shows that, for a signal with a period *T* the frequency of the signal is 1/*T* Hz. The signal frequency is then represented in the frequency domain as a single vertical arrow at that frequency, where the amplitude of the arrow represents the amplitude of the signal.

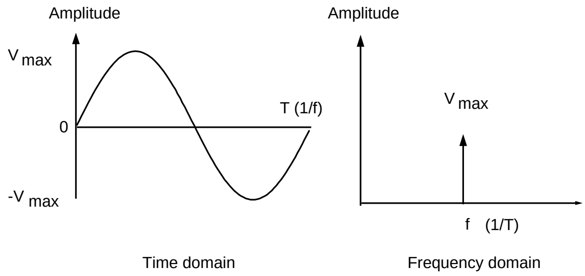

Figure 2.1 Representation of signal in frequency and time domains

2.3 Bandwidth

In general, in a communication system, bandwidth is defined as the range of frequencies contained in a signal. As an approximation, it is the difference between the highest and the lowest signal frequency, as illustrated in Figure 2.4. For example, if a signal has an upper frequency of 100 MHz and a lower of 75 MHz, the signal bandwidth is 25 MHz. Normally, the larger the bandwidth the greater the information that can be sent. Unfortunately, normally, the larger the bandwidth the more noise that is added to the signal. The bandwidth of a signal is typically limited to reduce the amount of noise and to increase the number of signals transmitted. Table 2.2 shows typical bandwidths for different signals.

The two most significant limitations on a communication system performance are noise and bandwidth. In a data communications system the bandwidth is normally defined in terms of the maximum bit rate. It can be shown that this can be approximately twice the maximum frequency of transmission through the system.

Table 2.2 Typical signal bandwidths

Application	*Bandwidth*
Telephone speech	4 kHz
Hi-fi audio	20 kHz
FM radio	200 kHz
TV signals	6 MHz
Satellite comms	500 MHz

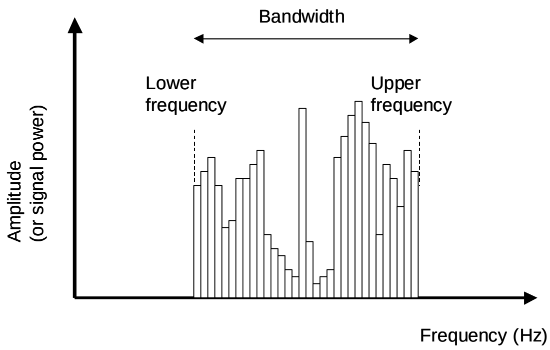

Figure 2.2 Signal bandwidth

2.4 Bandwidth requirements

The greater the rate of change of an electronic signal, the higher the frequencies that will be contained in its frequency content. Figure 2.5 shows two repetitive signals. The upper signal has a DC component (zero frequency) and four frequencies, f_1 to f_4. Whereas the lower signal has a greater rate of change than the upper signal and it thus contains a higher frequency content, from f_1 to f_6.

Typically, in a cascaded system, the overall bandwidth of the system is defined by the lowest bandwidth of the cascaded elements. Digital pulses have a very high rate of change around their edges. Thus, digital signals normally require a larger bandwidth than analogue signals. In a digital system made up of cascaded elements, each with its own bandwidth, the overall bandwidth will be given by the lowest bandwidth element (as defined in bps), as illustrated in Figure 2.6. This changes if there are parallel channels, as the bandwidth capacity of a parallel route is equal to the sum of the two parallel routes. For example, if data could take two channels, each with a bandwidth of 1 Mbps, the total bandwidth would be 2 Mbps, as 1 Mbps could flow over each channel (assuming that the data can be split between the two streams). Typically, bandwidth is not dedicated to a single connection, and must thus be divided by several connections. For example, if the maximum bandwidth of a channel is 10 Mbps, and that this is split between 10 users. If each of the users were equally using the channel then the bandwidth available to each user will be 1 Mbps. This type of equal sharing may not be the best solution, thus, in some cases users may be allocated a given share of the allocation.

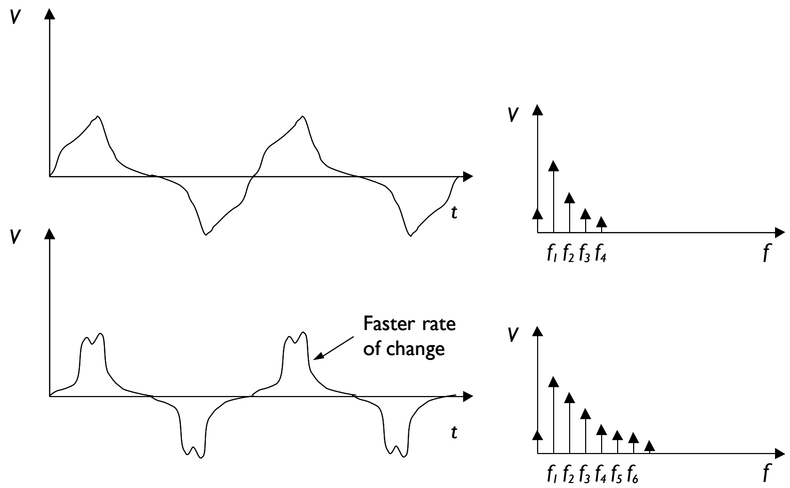

Figure 2.3 Frequency content of two repetitive signals

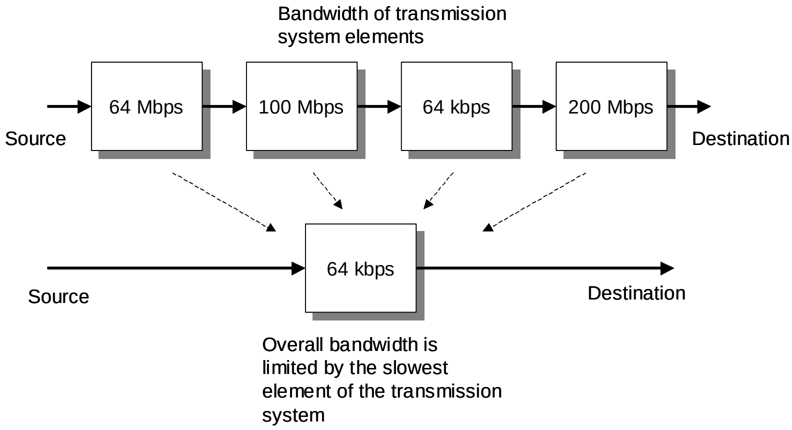

Figure 2.4 Overall bandwidth related to the system bandwidth elements

2.5 Noise and signal distortion

Noise is any unwanted signal added to information transmission. The main sources of noise on a communication system are:

- **Thermal noise.** Thermal noise occurs from the random movement of electrons in a conductor and is independent of frequency. The noise power can be predicted from the formula:

  $$N = k\,T\,B$$

  where N is the noise power in watts, k is Boltzman's constant (1.38×10^{-23} J/K), T is the temperature (in K) and B the bandwidth of channel (Hz). Thermal noise is predictable and is spread across the bandwidth of the system. It is unavoidable but can be reduced by reducing the temperature of the components causing the thermal noise. Many receivers which detect very small signals require to be cooled to a very low temperature in order to reduce thermal noise. A typical example is in astronomy where the temperature of the receiving sensor is reduced to almost absolute zero. Thermal noise is a fundamental limiting factor of any communications system.
- **Cross-talk.** Electrical signals propagate with an electric and a magnetic field. If two conductors are laid beside each other then the magnetic field from one couples into the other. This is known as crosstalk, where one signal interferes with another. Analogue systems tend to be affected more by crosstalk than digital ones, but noise in a digital system can could severe errors, if the noise is large enough to change a 0 to a 1, or a 1 to a 0.
- **Impulse noise.** Impulse noise is any unpredictable electromagnetic disturbance, such as from lightning or from energy radiated from an electric motor. It is normally characterized by a relatively high energy, short duration pulse. It is of little importance to an analogue transmission system as it can usually be filtered out at the receiver. However, impulse noise in a digital system can cause the corruption of a significant number of bits.

A signal can be distorted in many ways, especially due to the electrical characteristics of the transmitter and receiver and also the characteristics of the transmission media. An electrical cable contains inductance, capacitance and resistance. The inductance and capacitance have the effect of distorting the shape of the signal whereas resistance causes the amplitude of the signal to reduce (and also to lose power).

2.6 Capacity

The information-carrying capacity of a communications system is directly proportional to the bandwidth of the signals it carries. The greater the bandwidth, the greater the information-carrying capacity. An important parameter for determining the capacity of a channel is the *signal-to-noise ratio* (SNR). This is normally defined, in decibels, as:

$$\frac{S}{N}(dB) = 10\log_{10}\frac{SignalPower}{NoisePower}$$

For example, if the signal power is 100 mW, and the noise power is 20 nW, then:

$$\frac{S}{N}(dB) = 10\log_{10}\frac{100\times10^{-3}}{20\times10^{-9}}\ \text{dB}$$

$$\frac{S}{N}(dB) = 10\times\log_{10}\left[5\times10^{6}\right]\text{dB}$$

$$\frac{S}{N}(dB) = 6.7\ \text{dB}$$

In a digital system, Nyquist predicted that the maximum capacity, in bits/sec, of a channel subject to noise is given by:

$$Capacity = B.\log_2\left[1+\frac{S}{N}\right]\quad \text{bits/sec}$$

where B is the bandwidth of the system and S/N is the signal-to-noise ratio. For example, if the signal-to-noise ratio is 10 000 and the bandwidth is 100 kHz, the maximum capacity is:

$$\begin{aligned} Capacity &= 10^5.\log_2\left(1+10^4\right)\ \text{bits/sec} \\ &\approx 10^5.\frac{\log_{10}\left(10^4\right)}{\log_{10}(2)}\ \text{bits/sec} \\ &= 13.3\times10^5\ \text{bits/sec} \end{aligned}$$

$$\log_x(y) = \frac{\log_{10}(y)}{\log_{10}(x)}$$

Attenuation is the loss of signal power and is normally frequency dependent. A low-pass channel is one which attenuates, or reduces, the high frequency components of the signal more than the low frequency parts. A band-pass channel attenuates both high and low frequencies more than a band in the middle.

The bandwidth of a system is usually defined as the range of frequencies passed which are not attenuated by more than half their original power level. The end points in Figure 2.7 are marked at 3 dB (the –3 dB point) above the minimum signal attenuation. Bandwidth is one of the most fundamental factors as it limits the amount of information which can be carried in a channel at a given time. It can be shown that the maximum possible symbol bit rate of a digital system on a noiseless, band-limited channel is twice the channel bandwidth, or:

Maximum symbol rate (symbols/sec) = 2 × Bandwidth of channel

If a signal is transmitted over a channel, which only passes a narrow range of frequencies than is contained in the signal then the signal will be distorted. This is illustrated in Figure 2.8, where the maximum frequency content occurs when the 10101… bit sequence occurs. The minimum frequency content of this bit pattern will be B Hz. The symbol bit rate will thus be twice the highest frequency of the channel. The reason that the rate is referred to as a symbol rate, and not a bit rate, is that the symbol rate can differ from the bit rate. This is because more than one bit can be sent for each symbol. This typically happens with modems, where more than one bit is sent for every symbol. For example, several amplitudes of symbols are to be sent, such as four amplitudes can be used to represent two bits. The symbol rate for speech-limited channels will be 8,000 symbols per second (as the maximum frequency is 4 kHz). As four bits can be sent for every symbol, the bit rate will be 32 kbps.

If the frequency characteristics of the channel are known, the receiver can be given appropriate compensatory characteristics. For example, a receiving amplifier could boost higher frequency signals more than the lower frequencies. This is commonly done with telephone lines, where it is known as channel equalization.

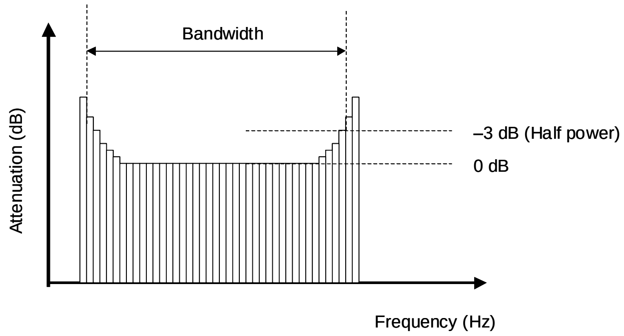

Figure 2.5 Bandwidth of a channel

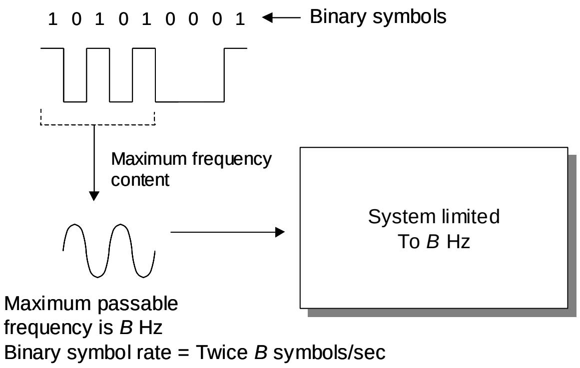

Figure 2.6 Maximum binary symbol rate is twice the frequency of the bandwidth